Polar synchronization and the synchronized climatic history of Greenland and Antarctica

Jeseung Oh*, Elizabeth Reischmann, José A. Rial

Wave Propagation Laboratory, Department of Geological Sciences, University of North Carolina, Chapel Hill, NC 27599-3315, USA

ABSTRACT

Stable isotope proxies from ice cores show subtle differences in the climatic fluctuations of the Arctic and Antarctic, and recent analyses have revealed evidence of polar synchronization at the millennial time scale. At this scale, we analogize the polar climates of the last ice ages to two coupled nonlinear oscillators, which adjust their natural rhythms until they synchronize at a common frequency and constant phase shift. Heat and mass transfers across the intervening ocean and atmosphere make the coupling possible. Here we statistically demonstrate the existence of this phenomenon in polar proxy records with methane-matched age models, and quantify their phase relationship. We show that the time series of representative proxy records of the last glaciation recorded in Greenland (GRIP, NGRIP) and Antarctica (Byrd, Dome C) satisfy phase synchronization conditions, independently of age, for periods ranging 1e6 ky, and can be transformed into one another by a \( \pi/2 \) phase shift, with Antarctica temperature variations leading Greenland’s. Based on these results, we use the polar synchronization paradigm to replicate the 800 ky-long, Antarctic, EPICA time series from a theoretical model that extends Greenland’s 100 ky-long GRIP record to 800 ky. Statistical analysis of the simulated and actual Antarctic records shows that the procedure is stable to change in adjustable parameters, and requires the coupling between the polar climates to be proportional mainly to the difference in heat storage between the two regions.

1. Introduction

The climate records seen in the polar ice cores can be characterized as nonlinear, complex, oscillating systems, with periods of abrupt warming separated by abrupt cooling. In order to gain a better understanding of the polar climate systems, and possibly even learn to predict the behavior of the polar climates, this paper seeks to propose a stable, reliable, and verifiable conceptual model of the polar climates. For this purpose, we demonstrate the effectiveness of modeling the polar climates as nonlinear oscillators, using the ice volume model (Saltzman, 2002) that represents each polar climate as simple Van der Pol oscillators. These oscillators are coupled and synchronized through the entire duration of the last glaciation.

Synchronization is a basic mechanism of self-organization in complex systems. Its presence in a system can be identified when two nonlinear oscillators (in our case, the polar climates) adjust their initially different natural frequencies to a common frequency with constant relative phase, also known as frequency entrainment and phase locking (Pikovsky et al., 2001). The oscillators must be nonlinear in order to respond to mutual forcing in such a way that their natural frequencies (and phases) are modified by their interaction. Possible evidence of other instances of synchronization within the climate system have been found in non-polar paleoclimate records as well, including the link between the Indian Monsoon and El Nino-Southern Oscillation (ENSO) (Lund and Mix, 1998). Higher frequency examples in recent climate records include the links between the Indian Monsoon and El Nino-Southern Oscillation (ENSO) (Maraun and Kurths, 2005; Tsonis et al., 2007) and the teleconnections between the North Atlantic, the Ethiopian Plateau and the Mediterranean (Feliks et al., 2010), among others. However, at this time, these connections have not been analyzed for their phase and frequency locking, so while they do demonstrate the transfer of energy necessary for synchronization to take place, synchronization itself has not yet been identified.

The possible connections between northern and southern polar temperatures have been identified on multiple occasions, all of which support modeling the polar climates as connected, nonlinear...
oscillators with the potential for synchronization. EPICA Community Members (2006) were first to identify a linear relationship between the stadial intervals at the poles during the MIS3 interval 50 ka–30 ka (1 ka = 1000 years) and then suggest that this trend extended to cover the last glaciation. Prior to this, Crowley had put forward the basic bipolar hemispheric seesaw hypothesis (Crowley, 1992; Broecker, 1998) as an explanation of how the abrupt warming episodes in the North Atlantic lead to the beginning of cooling episodes in Antarctica, namely via polar climate communication through meridional (equatorially asymmetric) heat transport and North Atlantic deep water (NADW) production. Blunier et al. (1998) and Blunier and Brook (2001) built on this idea, demonstrating that events in Greenland’s climate follow those in Antarctica by about 1–3 ky (1 ky = 1000 years) and that this is due to the ocean controlling the climate at both poles. Hinnov et al. (2002) studied the specific connection between the Byrd and GRIP2 records’ inter-hemispheric anti-phasing (180° phase shift) of the Dansgaard–Oeschger (DO) oscillations over the 10–90 ka interval. Even more recently, Steig (2006) reported a π/2 phase shift between the polar climates, seen by analyzing high-resolution records from EDML (Antarctica) and NGRIP (Greenland) cores. Barker et al. (2009) then published data from the South Atlantic which demonstrated the existence of rapid but opposite temperature changes occurring at the same time as those documented in the north and proposed a link between the DO oscillations in the Arctic and the sub-Antarctic temperature variations. In a follow up study, they used Crowley’s simple, conceptual bipolar seesaw model to forecast the unknown Greenland record using the 800 ky record of Antarctica (Barker et al., 2011), though the lack of actual Greenland records beyond ~120 ka means that they were unable to validate their results. Finally, in response to this, Rial (2012) proposed the nonlinear phase synchronization of the millennial-scale polar climates fluctuations during the last glaciation as an explanation for the apparent teleconnection between the Polar Regions.

While all the studies mentioned above serve as a basis of justification for this paper, they have all either lacked a statement of quantified statistical significance of the relationship between the two polar regions, only extracted millennial frequency band components using purely linear decomposition techniques, or focused on isolated, short events over the last glacial period. The present study investigates the statistical significance of the presence of polar synchronization over the last glacial period using a data adaptive decomposition technique, surrogate data tests, and long-term methane-matched age models when possible, providing a more complete characterization of the polar connection and behavior of the polar climates as oscillators. Our results will show that the polar behavior is well represented by the polar synchronization model throughout the 800 ky interval of recorded polar climate history. The approach demonstrated in this paper uses the aforementioned, simplified, climate model to closely reproduce the 100 ky Greenland ice core record (GRIP), then extends it a further 700 ky in the past, creating a 800 ky-long simulation of the Greenland δ18O time series. This reconstructed Arctic signal is then numerically transformed into an 800 ky-long simulation of Antarctica’s EPICA temperature proxy via the assumption of synchronization. We show that bi-directional, or mutual, phase synchronization affects the millennial cycles but does not appear to affect the long periods dominated by the Milankovitch forcing. A more detailed discussion of the role of nonlinear phase synchronization of the climate system to Milankovitch forcing over the last 5My is the topic of a recent paper by our research group (Rial et al., 2013).

Before exploring the modeling process and our results, though, it is important to emphasize that, throughout this paper, the use of the term synchronization or phase synchronization refers to frequency entrainment and phase lock (i.e. a constant phase difference), and so does not relate to the term asynchronous as used by some authors to refer to the polar climate being locked out of phase (Blunier et al., 1998; Stenni et al., 2010). Also, polar synchronization is unrelated to the technique of synchronizing age models using methane records from both Polar Regions (e.g., Blunier and Brook, 2001), though we have indeed made use of the methane-matched age models to compare the phases of Byrd and GRIP and to tune NGRIP and Dome C records to each other (details will be explained later with Fig. 1). This precondition allowed us to analyze phase differences between the two time series assuming the differences in age models to be small and likely irrelevant.

Our paper begins with an outline of the specifics of polar synchronization and describes how to transform one polar climate proxy record into the other in Section 2. Section 3 shows the model simulation of 800 ky of Antarctic climate record through the transformation of Greenland’s using both data and the model, as well as discussing model stability under parameter change. Section 4 includes a brief discussion of results, followed by conclusions in Section 5. Details of the mathematical models and statistical data analysis are described in the Appendix.

2. Polar synchronization paradigm

2.1. Polar synchronization

In order to demonstrate the utility of polar synchronization as a model, one must begin with an understanding of synchronization and why it is possible to say that the poles demonstrate its presence. Synchronization itself is a well-established phenomenon, first proposed via Christian Huygens’ classic observations of synchronized clocks (Huygens, 1673). Comparing these slightly nonlinear, weakly connected clocks to our more strongly nonlinear, still weakly connected polar climates makes his theory a direct analogy to explain the linked behavior of the polar climates. Huygens stated that two clocks with initially different frequencies (in our case, the poles) hanging from the same beam (connected by oceans and atmosphere) eventually synchronize to a common frequency and phase lock (Pikovsky et al., 2001; Bennett et al., 2002), even with very weak coupling. The phase lock can either be in-phase, anti-phase, or an arbitrary, constant phase. This lock can be observed throughout paleoclimate time series from the polar regions by inducing a constant phase shift of π/2 to the time series of the most representative abrupt climate events during the last glaciation recorded in Greenland and Antarctica. All millennial scale frequency components in the signal are π/2 shifted, meaning each of these frequency components is shifted by one-fourth of its period. Thus time series pairs like NGRIP-DomeC or GRIP-Byrd can be formally described as approximate Hilbert transform pairs (Bracewell, 1986; see Appendix A). This process (Figs. 1–3) will be more thoroughly explained in the second part of this section; however, it is important to note this characteristic phase lock here in order to better demonstrate the parallel between Huygens’ clocks and our polar climate records.

Unlike the clocks, though, climate oscillators are often highly nonlinear, multidimensional and chaotic, which allows for the presence of many forms of synchrony. For instance, as coupling is increased from zero, nonsynchronous states will undergo phase synchronization (PS), as appears to occur in the polar paleoclimate records. PS does not involve or require amplitude correlation. Other types of oscillations, though, require that the two linked oscillators be systems of identical oscillators, including for what is known as lag and complete synchronization. In identical systems, when coupling is increased beyond what is needed to induce PS, it produces lag synchronization (LS), which appears as
a coincidence of two systems $x_1(t)$, $x_2(t)$ shifted in time so that $x_1(t + \tau) = x_2(t)$. Increasing the coupling further can cause the time shift ($\tau$) to decrease and the oscillations in interacting systems to become completely identical, $x_1(t) = x_2(t)$, a state which is called complete synchronization (CS) (Rosenblum et al., 1997). Here, even the slightest difference between the oscillators leads to the eventual disappearance of the exact coincidence in their phase trajectories (Balanov et al., 2009). The final type of identified synchronization is generalized synchronization (GS), which explains synchronization between an autonomous (not forced) driving system and a response system. Since the driving system is not affected, the connection is named uni-directional. GS can be defined simply as the presence of some functional relation between the response (slave) and the driver (master). The driven system must be nonlinear (Rulkov et al., 1995; Rosenblum et al., 1997).

Being able to identify any type of synchronization in the ice cores, though, is a complicated process, given that each ice core is affected by a variety of factors other than temperature, making identification of direct interactions difficult. In addition to this, uncertainties in dating can also complicate phase analysis between cores. In spite of these problems and the number of possible synchronization types, our analysis suggests that a few simple rules controlling the relationship between polar climates do exist, allowing us to simulate the 100 ka–800 ka interval in Greenland, and use it to successfully reproduce the known last 800 ky of climate fluctuations in Antarctica, thus validating the significance of the simulation from the synchronization model.

2.2. Transforming one polar climate into the other

While we briefly mentioned the adherence of polar paleoclimate record data to the necessary phase lock in the previous section, the process of establishing a quantified phase separation of two proxies is more complicated than just a simple addition of phase. Detecting phase synchronization, as is illustrated in Figs. 1, depends highly on an accurate determination of the phases of each oscillator. In the case of the polar climates, which are not the result of a simple function, the process of establishing good instantaneous phase values for comparison must be tackled in several steps.

To start, age model matching by methane records (e.g. Blunier et al., 1998; Capron et al., 2010) allows for the comparison of phases of the polar proxy records, as any uncertainty in dating can lead to distortions of phase and frequency alignments. In order to avoid this, the methane-matched ages of GRIP (Greenland, black line in Fig. 1A) and Byrd
Antarctica, black line in Fig. 1B) are set as standard age models. We then used a Monte Carlo approach (e.g., adapted from Blunier et al., 2007) to find a common age model for the previously independently dated North-South pair of NGRIP-Dome C (originals seen in blue, Fig. 1A and B respectively). Through the age matching procedure, we obtained age-matched NGRIP and Dome C records (red lines in Fig. 1A and B respectively), which can then be tested for polar synchrony.

In order to determine the instantaneous phase of these now comparable time series, analytical functions are constructed for each time series (see Appendix A), a method that is commonly used for series generated by nonlinear oscillators (Gabor, 1946). The ice core temperature proxy signals, however, are not mono-components (a mono-component represents a simple signal involving only one oscillatory mode at any time instance) of a more complex signal, but rather are complex signals themselves; applying the Hilbert transform (HT) to such broadband signals can produce inaccurate results (Cohen, 1995; Huang et al., 1998). It is therefore important to first extract the mono-components of the signal using data-adaptive methods. Here we use the Empirical Mode Decomposition (EMD) (Huang et al., 1998) which provides Intrinsic Mode Functions (IMF) each of which is a mono-component. The Hilbert Transform (HT) is then applied to each mono-component signal, a procedure equivalent to a Hilbert Huang Transform (HHT) (Huang et al., 1998). To obtain the phase shifted signal, we applied HHT (equivalent to $\pm \pi/2$ shift) to the Dome C record with integral limits from past to future (see Appendix A). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

![Figure 2](image_url)

Fig. 2. (A) Polar synchronization ($\pi/2$ phase lock) between the North and South climate proxy records from Greenland (NGRIP) and Antarctica (Dome C). The age models of Dome C and NGRIP records are matched, i.e., they are tuned each other (see text for details). The Hilbert transform (HT) of Dome C correlates with NGRIP, which implies that the records of the Polar Regions are synchronized. (B) 3rd to 6th Intrinsic Mode Functions (IMFs) from NGRIP (black) and Dome C (red) records. Here we use the Empirical Mode Decomposition (EMD) (Huang et al., 1998) which provides Intrinsic Mode Functions (IMF) each of which is a mono-component. (C) Same as (B) but with Dome C signal transformed by $-\pi/2$ phase shift (Hilbert transform). The HT is applied to each mono-component signal, a procedure equivalent to a Hilbert Huang Transform (HHT) (Huang et al., 1998). To obtain the phase shifted signal, we applied HHT (equivalent to $\pm \pi/2$ shift) to the Dome C record with integral limits from past to future (see Appendix A). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
In both the frequency domain and for positive frequencies, the above transformation is equivalent to shifting each frequency component of the spectrum of the Antarctic record by \(-\pi/2\) radians which is mathematically equivalent to a Hilbert transform. Conversely, the inverse Hilbert transform of the Greenland record closely reproduces the Antarctic one, which is equivalent to shifting each positive spectral component by \(+\pi/2\) radians (the convention of time increasing from right to left means that a positive phase shift is a shift to earlier times). As seen in Fig. 2, the 1 ky–6 ky period signals are synchronized with \(\pi/2\) phase shift. Therefore, the Antarctic climate leads that of Greenland by 250 yr – 1500 yr. Hinnov et al. (2002) showed a time lead of BYRD DO oscillations over GISP2 by 384 ± 70 yr, which is within that range. This implies a frequency-dependent time lag. However, our observations indicate that the phase shift is frequency independent, as all Fourier components are shifted by \(\pi/2\), at least in the period band 1 ky – 6 ky.

The results of this decomposition and transformation can be seen in Fig. 2, which demonstrates the relationship between NGRIP and the HT of Dome C records. EMD is applied to both of the records (Fig. 2B) then the HT is applied to the selected IMFs of the Dome C record (Fig. 2C). The two pairs of records have been synchronized previously. As a result, the elements within each pair form an approximate Hilbert transform pair with a constant phase lock, meeting the condition for phase synchronization (see Appendix A). As shown in Fig. 3, the calculated phase lock is nearly constant and equal to \(\pi/2\) modulo \(2\pi\). The statistical significance of the phase lock at \(\pi/2\) is discussed in Appendix A.

Sudden phase jumps of \(2\pi\), which occur during short desynchronization episodes (Fig. 3), are likely excited by noise, timing errors or external forcing. As these transient effects rapidly disappear, phase synchronization resumes, showing continued influence of the link in spite of these abrupt discontinuities.

3. Simulation of Antarctic climates from Greenland climates

3.1. Frequency band separation

While this section is devoted to beginning to build and verify our model, the length of our data makes it necessary to take a closer look at the characteristics of our data series, as the scale here allows for multiple physical processes to be part of creating the oscillations seen in the polar records. Looking specifically at the 800 ky Dome C records, the spectra shows two disjointed frequency bands, one with periods shorter than \(\sim 50\) ky and one for periods longer than \(\sim 50\) ky. The 800 ky-long EPICA proxy records (and most temperature proxy records for the last million years) also have a well established tendency of consisting of easily identifiable and disjointed groups of statistically significant power peaks in the long period band (>70 ky) and the short period band (<45 ky), with little or no interaction between the two. To proceed, we split the record along this line and treat them as two separate records: one that is assumed to reflect the high frequency response of sea ice, and one that reflects the long period response of the major ice sheets to eccentricity-induced changes in insolation. This separation is necessary, as we simulate these two behaviors with different simplified climate models since they seem to derive from different physical processes. In the end, the two model records are added to generate a simulated EPICA Dome C time series.

However, separating frequencies of what may be non-linearly generated, non-stationary time series requires careful
consideration of the possibility that the long and short period responses are connected, violating superposition. To ensure that the separation is valid, the data adaptive decomposition methods for non-linear and non-stationary time series Empirical Mode Decomposition (EMD) and Singular Spectrum Analysis (SSA) were used, as well as a conventional linear Butterworth filter for comparison of a linear process with these nonlinear methods (Fig. 4).

Specifically, the Butterworth filter outputs a linear combination of the values of the components in the input signal. This linear filter is a simple algorithm that computes a weighted average of the neighborhood components. Note that the EMD and SSA techniques have been broadly adopted for the analysis of paleoclimatic time series (e.g., Gloersen and Huang, 2003; Lin and Wang, 2006; Sole et al., 2007; Huang and Wu, 2008) and are described in detail in Appendix D.

Going back to separating the frequency bands, we return to considering Dome C, shown in Fig. 4A, and its long period components extracted using EMD, SSA and the Butterworth filter. The decomposed long period signals show that separation using a linear filter is not substantially different from using the more specialized nonlinear filters. Fig. 4B shows the high frequency components (shorter than ~50 ky) extracted by EMD, SSA, and a linear filter, which again show very little distortion after linear filtering. This suggests a true separation of the records of the processes that produce the long and short period signals in both a linear and nonlinear combination.

3.2. Modeling the climate of the South from that of the North

In climate science, there are few well-established, simplified models that include both aperiodic forcing and stochastic effects, as is needed to model both the long and short frequency sections of the Antarctica records described above. On the other hand, experience in many areas of science indicates that relaxation oscillators are excellent candidates for the simulation of natural nonlinear oscillators and thus should be useful in paleoclimate studies. The Van der Pol oscillator is the most common relaxation model applied to various fields of climate research. Saltzman and Moritz (1980), Saltzman et al. (1981), Saltzman and Sutera (1984) and later Saltzman (2002) formally introduced a set of nonlinear, ordinary differential equations for sea ice extent and average ocean temperature in a glacial atmosphere equivalent to a van der Pol/Duffing oscillator. Elements of their model were later incorporated in models of interdecadal-millennial oscillations by Yang and Neelin (1993); Zhang et al. (1995); Egger (1999). Rial and Yang (2007) showed how frequency modulation could be easily demonstrated in the Greenland proxy data using a Van der Pol oscillator. Its computational simplicity, the fact that its oscillations are robust against perturbation and can be tuned to simulate the slow build up followed by the sudden release observed in many climate records have made the Van der Pol oscillator popular among paleoclimate modelers, including Stommel (1961), Källén et al. (1979), Paillard (1998), Schulz et al. (2004), Colin de Verdière et al. (2006),

Fig. 4. (A) Long period signals obtained by various filtering methods. Black line on the top shows Epica Dome C temperature proxy. a) Long period signal obtained by decomposing the data into 13 IMFs. The long period signal shown is calculated by summing 8th–12th IMFs. The IMFs are selected so that they produce ~100 ky frequency band signal. b) Long period signal obtained using SSA. For the long period from SSA, we adopted first 3 components with 1600 point windows. c) Low pass filtered signal using a conventional linear filter. All three techniques extracted essentially the same wave shape of long period signal from the data. See Appendix for details of the filtering methods. (B) High frequency signals after removal of long period components in (A).

Therefore, to simulate the high frequency response, we use the basic Van der Pol Synchronization Oscillator (VSO) model, which describes each polar climate as a nonlinear Van der Pol oscillator and couples them with both reactive and dissipative coupling (see Appendix B). The VSO model consists of two pairs of first order, ordinary differential equations that represent two coupled Van der Pol oscillators, with each pair describing sea ice extent and mean ocean temperature at each Polar Region. The model is externally forced by summer insolation at 65 degrees North.

Rial (2012) used a VSO to simulate temperature proxies in short segments of the Greenland and Antarctica records and assess the presence of synchrony between the Polar Regions. Here we extend that study, employing a VSO to simulate the Antarctic temperature proxy record from that of Greenland (Fig. 5). Fig. 5A shows the theoretical Greenland temperature (calculated with a single Van der Pol model oscillator) over the last 400 ky compared to the EPICA record. Fig. 5B shows the modeled southern temperature oscillations with external forcing and noise but without synchronization ($q_1 = q_2 = 0$; see Appendix B for the meaning of the parameters). As synchronization is incorporated, the VSO model produces a far better simulation of the data (Fig. 5C). Comparison of Fig. 5B and C suggests that the effects...
of noise and insolation forcing are nearly suppressed when the coupling is applied, though the specific role of all parameters in the model will be discussed in detail in the next section.

Fig. 5D shows the simulation results using the polar synchronization VSO model. From 100 ka to 800 ka, this model is based upon the theoretical northern temperature variations depicted in Fig. 5A.

Defining and modeling the longer frequency section of the record is slightly more complicated. After all, while the ∼100 ky cycles describing the predominant glacial–interglacial period since ∼1.2 Ma (at the Mid–Pleistocene Transition, MPT) are clearly recorded in most paleoclimate proxy data sets, the cause of the unexpected strength of these climatic fluctuations remains elusive, in spite of vigorous research over the last few decades. Recently, researchers have suggested that orbital insolation forcing is not directly responsible for the 100 ky cycles. Instead, alternate hypotheses to explain the trigger of these 100 ky cycles include internal oscillations in the climate–cryosphere (Gildor and Tziperman, 2001), oscillations of the climate–cryosphere–carbonosphere system (e.g. Saltzman and Maasch, 1988; Paillard and Parrenin, 2004), each second or third obliquity cycle (Huybers and Wunsch, 2005), each fourth or fifth precessional cycle (Ridgwell et al., 1999), and a combination of the precessional and obliquity components of the orbital forcing (Paillard, 1998). However, most simulated 100 ky cycles based on the above theories exhibit weaker amplitude than those actually observed in paleoclimate proxies, as well as strong variability at the ∼1/400 ky eccentricity frequency, and thus do not account for the observed changes in ice volume (Ganopolski and Calov, 2011). However, Rial et al. (2013) suggest that the ∼100 ky signal is mostly an internal oscillation, frequency modulated by the eccentricity cycles. Here, it is proposed that master-slave synchronization of the internal oscillation to the eccentricity forcing (a form of resonance) accounts for the strength of the 100 ky glacial–interglacial cycles.

In order to simulate the separated long period response, we use the ice volume model devised by Saltzman (2002), mentioned earlier and described in detail in Appendix C. This model produces free oscillations with periods near 100 ky of ice mass, carbon dioxide, and deep ocean temperature with appropriate parameter...
values in the absence of external (astronomical) forcing. Saltzman (2002) indicated the region for the parameters in which the model produces stable oscillations and provided the simulation of $w_{100}$ ky period glacial cycles with those parameter values (Saltzman, 2002). As shown in Fig. 6 the ice sheet model closely reproduces the frequency and amplitude of glacial cycles with the simulated 413 ky forcing of the past 800 ky.

Now that we have developed simulations describing both the long and short period frequency bands of the records, they can be combined to create a complete model for the last 800 ky. Fig. 6C shows the long period component from the ice sheet model added to the simulated high frequency signal.

3.3. Model stability under parameter uncertainty

There are three sources of uncertainty generally considered in a modeling procedure: the stochastic nature of a given model, parameters, and observed input data. Since the stochastic nature of the VSO model is in the stochastic term of the model and the model does not require observed data, the uncertainty of the VSO modeling results arises from the uncertainty in the values of the three adjustable parameter pairs (see Appendix B) $a_i$, $u_i$, and $q_i$ ($a_1$, $u_1$, $q_1$ in the North; $a_2$, $u_2$, $q_2$ in the South), namely the strength of external forcing, the assumed natural oscillating frequency, and the coupling strength, respectively. Here, we investigate the effect of parameter uncertainty on the model stability and provide the range of the parameters within which the uncertainty of the parameters is restricted to be under that from the stochastic term. First, we find the parameter set that provides the best fit according to the correlation coefficient ($R$) and root mean square error (RMSE); then we assume that the parameter set is true. To explore the uncertainty from the stochastic term, we ran the model 10,000 times, then calculated the corresponding $R$ and RMSE with the fixed true parameters so that the model responded only to internal stochastic forcing. From the 10,000 values, we determined 5% and 95% of $R$ (0.43, 0.57) and RMSE (0.85, 1.29) as lower and upper limits respectively. Then, as a parameter changes, if both $R$ and RMSE remain within the interval 95% and 5% respectively, we consider the model stable to changes in that parameter (Fig. 7A). With this information, we obtained the regions of each parameter space within which the VSO model simulates a temperature proxy in Antarctica compatible with the observations, which is shown in Table 1 and where the parameter uncertainty is restricted to smaller than the uncertainty from the stochastic term.

Fig. 7 shows the model uncertainty from the parameters. The model behavior according to the parameter uncertainty and the ranges of the parameters that produce smaller effects than stochastic forcing can be summarized as follows:

i) Insolation parameters ($a_i$, $q_i$), Fig. 7A: Parameters $a_i$ regulate the strength of external forcing. The synchronization model originally included two forcing terms ($a_1$, $a_2$) at each pole (Eqs. (B.1) and (B.3)), but we use only one external forcing term ($a_2$) in the south for this simulation. Strong external forcing
overwhelms the model’s responses to the other parameters ($q$ and $\omega$), thus $a_2$ has to be constrained under a certain value because when external forcing is too strong, the simulated time series just follows the shape of forcing. From Fig. 7A, calculated as $a_2$ increases, the model produces stable simulation when $0.05 < a_2 < 0.10$.

ii) Oscillation parameters ($\omega_1$, $\omega_2$), Fig. 7B: The angular frequencies $\omega_1$ and $\omega_2$ are given by $\omega_1 = 2\pi/TN$ and $\omega_2 = 2\pi/TS$ where $TN$ and $TS$ are the assumed periods of the natural oscillations of the North and South climates respectively.

The model produces close simulations when the periods are within both the millennial scale and the range of the assumed period of the thermohaline circulation (Marotzke et al., 1988; Bond et al., 2001). Fig. 7B shows the range of the natural frequency parameters which produces smaller uncertainty and a close simulation of Antarctic Records (e.g. TN: 1500yr and TS: 3000yr or vice versa).

iii) Coupling parameters ($q_1$, $q_2$), Fig. 7C: RMSE ranges from 0.86 to 1.26 but these values fall within the range of RMSE values generated by the stochastic response. Thus, RMSE cannot be used as a criterion for selecting best parameter values in this case. The results show that the correlation coefficients dramatically decrease as $q_2$ increases and the model produces stable outputs when $q_1 > 60$ and $q_2 < 10$. Consequently, the reactive coupling term ($q_2$) can be dropped in modeling while the dissipative term ($q_1$) is required to be strong for both smaller uncertainty and a close simulation of the Antarctic temperature proxy.

4. Discussion

We have shown that the time series of temperature proxies from the Polar Regions form (approximately) Hilbert transform pairs, and thus are orthogonal (Bracewell, 1986) to each other; much like sine and cosine these climate records are in quadrature. The physical processes behind this demonstrated fact are not understood at this time, but our current research efforts point to the thermohaline circulation as the most likely driver of the climate connection between the poles. We have also shown that a synthetic record of Greenland’s temperature variation can be computed to theoretically extend the record to 800 ka and then used successfully, at the millennial scale, to estimate the Antarctic temperature variations recorded in the EPICA core, on the assumption of polar synchronization. This extends the presence of polar synchronization and $\pi/2$ phase shift to at least 800 ky BP. The models used are robust with respect to changes in the adjustable parameters and suggest that the connection between northern and southern climates is strongest when it involves the dissipative properties of the ocean/atmosphere, namely the difference between northern and southern heat flux.

The simplified VSO model is purely heuristic and does not have enough detail to provide a more specific view of the systems that couple the two Polar Regions. On the other hand, the idea of climatic synchronization is likely to be general, and occur in all time scales. Indeed, modern patterns of atmospheric/oceanic circulation of large spatial scale and a high degree of temporal persistence may be examples of synchronization. For instance, the North Atlantic Oscillation (NAO) is two alternating spatial patterns of atmospheric pressure in the North Atlantic and the Arctic, which may oscillate due to a synchronization mechanism. Also, ENSO could be understood as a synchronized pattern of oscillating temperature and atmospheric pressure that alternates between the two ends of the tropical Pacific basin. In the case of polar synchronization, given the earlier definitions of different synchronization and the differences in the two poles as oscillators, we could hypothesize that the polar climates were once unrelated to each other and likely chaotic, and that, through weak oceanic/atmospheric coupling, the two polar systems eventually synchronized by slowly modifying their own frequencies and phases to respond to each other’s influence, until synchronization ensued and stabilized.

5. Conclusions

This study presents statistical investigations to explore and quantify the long-term relationship between Greenland and Antarctic’s climate variability with polar synchronization as a mechanism to connect them, as well as presenting simplified, easily computable, and stable models for the last 800 ky of the polar paleoclimates. Using methane–matched records from GRIP and Byrd ice cores, we were able to match the age models of NGRIP and Dome C ice core records through a Monte-Carlo approach. The age-matched records not only showed a statistically significant, synchronized relationship with a $\pi/2$ phase shift, but also specifically showed that polar synchronization is supported by statistical analysis using EMD and RSS tests. Finally, we are able to conclude that polar synchronization has remained constant for millennial-scale climate oscillations over the last 100 ky, and likely over the last 800 ky, which may indicate that the driver of variability is internal to the system, likely the Atlantic thermohaline circulation. Using the polar synchronization paradigm, we successfully simulated Antarctic temperature records for the last 800 ky as deduced from synthetic Greenland records created using the VSO model. The VSO model consists of only a few terms, which describe relaxation oscillation, coupling effects, external forcing, and noise. Analysis of the model uncertainty supports that simulated results are robust under small change of parameters. Use of this model and the simplified view of a highly complex system would be helped by further exploration of the underlying physical processes of the polar climates. However, synchronization as a paradigm for the linking of polar climate processes is well supported by our results and takes us a step further in our understanding of large-scale climate dynamics.

Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>60 – 6000</td>
<td>Rate of mean ocean temperature change</td>
</tr>
<tr>
<td>$q_2$</td>
<td>0</td>
<td>Mean ocean temperature change</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0</td>
<td>External forcing to north</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.05 – 0.12</td>
<td>External forcing to south</td>
</tr>
<tr>
<td>TN</td>
<td>750 – 4000</td>
<td>Oscillation (north)</td>
</tr>
<tr>
<td>TS</td>
<td>750 – 4000</td>
<td>Oscillation (south)</td>
</tr>
</tbody>
</table>

$a_1$ and $a_2$ adjust strength of external insolation forcing ($a_1$: north; $a_2$: south). These parameters modulate how strongly the insolation affects the natural oscillation of the climate. (a) The parameters are large enough: simulated results from the VSO model are completely dependent on the insolation. (b) These parameters are too small: the simulated results are independent of the insolation. Temperature fluctuation at millennial scale should be between (a) and (b). Though it is impossible to calculate the exact amount of insolation affecting the millennial scale climate oscillation, we suggest the range of parameter values that could produce compatible simulation with the observation. $q_1$ and $q_2$ adjust strength of coupling effects ($q_1$: dissipative, $q_2$: reactive). These parameters modulate how strongly the north–south coupling affects to the simulated results. If both these parameters are zeros, north and south are simulated independently without synchronization. $q_1$ adjusts the strength of the coupling effect caused by differences in mean ocean heat flux which is calculated by differences between gradient of mean ocean temperature in north and south. $q_2$ adjusts the strength of coupling effect caused by differences in mean ocean temperature. The model simulation suggests that the synchronization occurs through differences in mean ocean heat flux rather than simple differences between two poles.
Acknowledgements

This research is supported by grants from the J. S. McDonnell Foundation (21st Century Science Initiative on Complex systems), and the National Science Foundation (Paleoclimate and P2C2 programs). All data used in this paper was provided by the National Snow and Ice Data Center, University of Colorado at Boulder, and the WDC-A for Paleoclimatology, National Geophysical Data Center, Boulder, Colorado.

Appendix

A. Synchronization and the π/2 phase shift

Determining the phase relationship between synchronized oscillators is performed by constructing analytic functions (Gabor, 1946) for each of the time series. Thus, the analytic function \( A(t) \) of the observed Antarctic temperature proxy time series \( a(t) \) is constructed as (Balanov et al., 2009):

\[
\Phi_A(t) = \arctan \left( \frac{H[a(t)]}{a(t)} \right); \quad \Phi_G(t) = \arctan \left( \frac{H[g(t)]}{g(t)} \right)
\]

where \( i = \sqrt{-1} \) and \( H[.] \) represents the Hilbert transform (HT) of its argument. The instantaneous frequency functions \( \omega_{AG} \) are, respectively,

\[
\omega_{AG}(t) = \left< \frac{d\Phi_{AG}(t)}{dt} \right>
\]

where the brackets represent time averages.

This approach is meaningful only if the embedded signal rotates around a fixed center (Maraun and Kurths, 2005) and this requirement is rarely satisfied for complex signals exhibiting variations on a wide range of frequencies. Therefore either suitable filtering is needed to eliminate the variations of the center or time derivatives of the signals could be used for the phase calculation (Osipov et al., 2003). Here, Empirical Mode Decomposition (EMD) performs the role of a suitable filtering. The centered rotations of the embedded signals from EMD are shown in Fig. A1. Consequently, we used the phase definition described by Eq. (A3) to calculate the phase difference between the decomposed Greenland and Antarctic signals, since the signals decomposed by EMD satisfy the condition.

Fig. A1. Embedding of decomposed DomeC records using EMD (3rd IMF to 6th IMF respectively). The decomposed signals rotate around a fixed center.
Synchronization of periodic self-oscillatory systems is defined as a phase entrainment
\[ \Delta \Phi(t) = |n\Phi_A(t) - m\Phi_C(t)| < \text{const}. \]  
\[ \text{(A.5)} \]
where \(n\) and \(m\) are integer numbers. In the presence of noise the phase difference is unbounded and performs a random-walk-like motion. If the noise is small, the frequencies are nearly locked, i.e., the relation between them is fulfilled on average:
\[ n \frac{d\Phi_A}{dt} = m \frac{d\Phi_C}{dt}. \]  
\[ \text{(A.6)} \]

To evaluate whether the phase synchronization defined in Eq. (A5) is just coincidence, we employed the significance test of rank-shuffled surrogate (RSS). This method generates surrogate data by randomly shuffling the rank of the original signals. For details of RSS, see Theiler et al. (1992) and Sun et al. (2012). We generated 10,000 surrogate pairs using RSS method. Fig. A2 shows the distribution of the phase differences in each data point from 10,000 surrogate pairs (same with Fig. 3A and B but using 10,000 pairs). Each box contains 10,000 lines corresponding to each pair. Though we could not visually see any peak from the boxplot, we did a statistical test to confirm whether the average number falling in the 3rd box (\(\pi/2\) phase difference) equals the number falling in the other boxes. The basic requirement in order to argue that the surrogate pairs show a \(\pi/2\) phase difference is that the 3rd bin has to have a peak. However, from the t-test (null hypothesis: the average number of points in the 3rd bin equals the average number of points in another bin), we could not reject the null hypothesis at 95% of significant level. Therefore, 10,000 surrogate pairs do not show \(\pi/2\) phase shift and we conclude that the detected phase synchronization is not an artifact.

**B. Van der Pol Synchronization Oscillation (VSO) model**

\[ u'_1 = -\omega_1^2 u_2(t) + a_1 M(t) + \xi(t) \]  
\[ \text{(B.1)} \]
\[ u'_2 = f_1(t, u) + q_1 \left[ u'_1(t) - u'_2(t) \right] + q_2 \left[ u_1(t) - u_2(t) \right] \]  
\[ \text{(B.2)} \]
\[ u'_3 = -\omega_2^2 u_4(t) + a_2 M(t) + \eta(t) \]  
\[ \text{(B.3)} \]
\[ u'_4 = f_2(t, u) + q_1 \left[ u'_3(t) - u'_1(t) \right] + q_2 \left[ u_3(t) - u_2(t) \right] \]  
\[ \text{(B.4)} \]
where \(f_1(t, u) = u_2(t) - (1/3)u_2(t)^3 + u_1(t), f_2(t, u) = u_4(t) - (1/3)u_4(t)^3 + u_3(t)\); \(\omega_1 = 2\pi/TN, \omega_2 = 2\pi/TS\).

Eqs. (B.1) and (B.2) describe mean ocean temperature \((u_1)\) and sea ice extent \((u_2)\) in the Arctic region and their interaction while Eqs. (B.3) and (B.4) describe Antarctic region \((u_3, u_4)\). Summer insolation at 65 degrees North \((M)\) externally forces the model, but the forcing intensity is adjusted by the different parameters \(a_1\) and \(a_2\) for Arctic and Antarctic region respectively. All these variables have arbitrary unit with zero-mean. The natural frequency of the oscillation is set up as \(\omega_1\) and \(\omega_2\), whose corresponding periods are calculated as \(TN = 2\pi/\omega_1\) and \(TS = 2\pi/\omega_2\) for Arctic and Antarctic region. The Gaussian white noise functions \((\xi(t)\) and \(\eta(t)\) with mean = 0, variance = 1) are applied to the both regions and the noise levels are set to be equal. Synchronization terms are included in Eqs. (B.2) and (B.4), which are assumed to be the difference in mean ocean heat flux (coefficient \(q_1\)) and difference in mean ocean temperature (heat storage) between two regions \((q_2)\). The employed synchronization form is generally adopted in these types of oscillators (Balanov et al., 2009).

**C. Ice volume (Saltzman) model**

The following equations describe the modified version of the ice sheet model (Saltzman, 2002).

\[ \frac{dX}{dt} = -X - Y - vZ \]  
\[ \text{(C.1)} \]
\[ \frac{dY}{dt} = -pZ + rY - sY^2 - Y^3 + aR(t) \]  
\[ \text{(C.2)} \]
\[ \frac{dZ}{dt} = -q(X + Z) \]  
\[ \text{(C.3)} \]
where \(R(t) = \cos(2\pi t/826 + e)\).

The model describes responses for ice \((X)\), carbon dioxide \((Y)\), and ocean temperature \((Z)\). Parameters \(p, q, r, s, v,\) and \(s\) are defined as structure determining parameters which adjust amplitude and frequency of the variables while \(v\) is function of others. \(R(t)\) is the external forcing term which is the rectified cosine function producing the 413 ky eccentricity signal.

**D. EMD and SSA**

Empirical Mode Decomposition (EMD) decomposes non-linear and non-stationary oscillatory time series into a number of Intrinsic Mode Functions (IMF), which is derived from the elimination of the oscillatory characteristics related with given resolution. Since IMFs illustrate partial Hilbert transforms of the time series, special features of smoothness in frequency and amplitude modulation are intrinsic in IMFs. The decomposition of the time series \(y(t)\) can be represented simply as Eq. (D.1) below, where, \(n\) is...
the total number of IMFs and \( r_n(t) \) is residue after extraction of \( n \) IMFs. \( r_1(t) \) generally describes a monotonic trend of the data, \( y(t) \) (For full description see, for example Huang et al., 1998; Huang and Wu, 2008.). Thus, we can write

\[
y(t) = \sum_{i=1}^{n} \text{IMF}_i(t) + r_n(t) \tag{D.1}
\]

The number of IMFs that EMD produces depends on the number of data points in the decomposed records. We have 10 IMFs from the EMD of NGRIP, Dome C, GRIP, and Byrd records. EMD decomposes the shortest component first (e.g. 1st IMF: the highest frequency component) and each frequency is assumed to contain a mono-component signal. The frequency band of each mono-component signal is extracted data adaptively while the frequency band extracted from Butterworth filter depends on a corner frequency defined by a user. Therefore, the justification of the selection of the decomposed frequencies (i.e. use of certain IMF) is not necessary. This also means that excluding some IMFs that have too short or long a period does not cause the problem of arbitrary selection for the frequency components. Here, we use the 3rd through 6th IMFs, which consist of millennial oscillations.

The main intent of Singular Spectrum Analysis (SSA) is to decompose the original time series into a sum of series that describe the major physical phenomena of the data. SSA also provides data-adaptive spectral filters and identifies noise characteristics as well as trends and periodic or quasi-periodic oscillations of the data. One of the important features of SSA is that the oscillatory patterns can be amplitude and phase modulated. The SSA technique comprises two steps: (1) decomposition consisting of embedding and singular value decomposition (SVD) and (2) reconstruction consisting of grouping and diagonal averaging. The theoretical and practical foundations of the SSA are described in Broomhead and King (1986), Vautard and Ghil (1989) and Golyandina et al. (2001).

References


